Remembering What Matters in Teaching and Learning of Mathematics

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Abstract: *In the wake of the COVID-19 pandemic in early Spring 2020, teachers moved quickly to distance and online learning. Unfortunately, this led some to revert to lecture- and worksheet-heavy instruction. In this paper, the authors remind us that online teaching and learning needn't be traditional or boring as they share approaches that encourage curiosity, creativity, and sense making.*

Keywords: *conceptual understanding, sense making, online teaching*

1 Introduction

Over the past few decades, the teaching and learning of mathematics has been transformed by the adoption of *Principles and Standards for School Mathematics* (NCTM, 2000) and *Common Core State Standards for Mathematics* (NGA Center and CCSSO, 2010). These documents promote relevant, focused, and targeted instruction that encourage teachers to actively engage students. Moreover, the standards encourage critical thinking as students acquire conceptual and procedural knowledge of mathematics. Indeed, it is a great time to be a mathematics educator! Or at least it was, until March 2020 when COVID-19 caused school-building closures and pushed educators across the country to the world of online and distance education.

Principles to Actions: Ensuring Mathematical Success for All (NCTM, 2014) describes a number of characteristics that mark effective instruction. These include (a) clear goals that provide a destination for students' learning path, (b) carefully selected tasks that lend themselves to various solution strategies and representations, and (c) meaningful discourse that enables students to share their thinking and connect strategies and solutions. Through explorations and discovery, students develop conceptual understanding of relevant procedures and algorithms. Teachers support students' productive struggle and thinking through purposeful questioning (Boston, Dillon, Smith, and Miller, 2017; Nolan, Dixon, Roy, and Andreasen, 2016; Nolan, Dixon, Safi, and Haciomeroglu, 2016; Smith, Steele, and Raith, 2017; Smith and Stein, 2011).

2 Impact of Closing School Buildings

Prior to COVID-19, many educators—supported by colleagues and professional organizations—had moved away from instruction marked by an 'I do, We do, You do' approach. A focus on procedural fluency was supplanted by greater emphasis on sense-making, comprehension, and application of concepts as students engaged in messy tasks, explored multiple approaches to solving problems, and spent significant amount of class periods engaged in discourse. Although students continued to practice procedures and apply algorithms, they were also provided with ample opportunity to engage in authentic mathematical activity.

In early 2020, as schools made their urgent shift to distance and online teaching due to the COVID-19 pandemic, many educators managed to adapt their instructional practices to the new environment. For some, unfortunately, the loss of a brick and mortar classroom resulted in a retreat from standards-informed instruction to didactic, lecture- and worksheet-heavy teaching, or at least the online version of it. The hum of a very strong math education community across our nation was no longer loud enough, no longer strong enough to sustain the instructional change that was needed to move our students forward in the learning of mathematics.

Online teaching and learning of mathematics is challenging, especially if instruction is to be delivered asynchronously. Some aspects of face-to-face instruction are more easily adapted to virtual classrooms than others. Development and application of procedures and algorithms are supported through instructional videos, allowing the "how" of mathematics to be adequately addressed. Unfortunately, it's not always clear how to foster critical thinking, problem solving skills, and engagement in online environments. Without adequate support, many educators have struggled to provide students with such opportunities. *What can mathematics educators ensure that students in virtual classrooms are making sense of mathematics and are immersed in problem-solving process?*

It is certainly possible to ignite student thinking and engage them in sense making online. Various resources support student learning with technology (e.g., Desmos). Teachers who incorporate manipulatives into daily instruction can continue to do so with virtual manipulatives. In addition, the mathematics education community has supported teachers by providing ideas and professional development on the design and delivery of engaging and rigorous lessons. Unfortunately, some households have limited technology access; some teachers are more tech savvy than others; and many feel pressure and stress to balance online instruction with daily life during the pandemic.

3 Preserving Effective Teaching and Learning of Mathematics

Let us step back. Note that it's not necessary to create elaborate interactive and technology-dependent lessons to support students' mathematical engagement or sense making. In the remainder of this paper, we present three time-tested approaches— 'Starting with the Answer', 'Changing Perspectives', and 'Choose Your Own Question.' These methods, rooted in effective teaching practices in brick-and-mortar environments, use and connect multiple representations, support productive struggle, build procedural fluency from conceptual understanding, and pose purposeful questions (NCTM, 2014). Our examples illustrate what can be done to support students in increasing their comprehension of mathematical concepts through inquiry, problem-solving, and critical-thinking in an online teaching and learning environment. The examples illustrate ways in which traditional lessons and tasks can be adapted to engage your students online.

3.1 Starting with the Answer

Many students experience mathematics sequentially. They are taught a problem-solving strategy, given problems to be solved using that strategy, then apply the strategy to find a correct answer. The 'Starting with the answer' method reverses this process—providing students with an answer and asking students to generate possible questions corresponding to the answer. This approach promotes reasoning, problem solving, and building procedural fluency from conceptual understanding.

Consider, for example, geometry standards targeting surface area and volume of solid figures. At the secondary level the emphasis is placed on the application of the surface area and volume formulas, rather than their memorization and recall. Therefore, instead of asking students to complete numerous computation problems involving finding surface area or volume of various solid figures, one can pose the following questions for students to reason through.

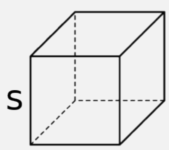
3.1.1 Task 1

The volume of a solid figure is 64 cubic units.

- What could be the dimensions of this figure?
- How many different figures could you identify that have volume of 64 cubic units?
- What are their dimensions?

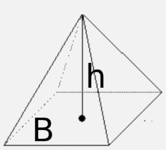
This task promotes reasoning since students have to make sense of the given information to solve the problem. First, students need an understanding of the concept of volume. Moreover, the task has multiple entry points. All students, regardless of mathematical background, can engage in the task at a level appropriate for them. Some students may perceive the figure as a simple cube (Figure 1); others, a rectangular pyramid (Figure 2), a cylinder (Figure 3), or another solid figure or combination of figures.

Figure 1: Cube with Volume of 64 cubic units



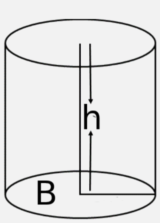
Formula for the volume of a Cube: $V = s^3$. Students do not necessarily need to know how to solve the equation: $s^3 = 64$. Simple substitution of values for s will lead them to the length of the side ($s = 4$ units).

Figure 2: Rectangular Pyramid with Volume of 64 cubic units



Formula for the volume of a rectangular pyramid: $V = \frac{1}{3}Bh$, with B the area of the base. Given that the volume of the pyramid is 64 cubic units, students need to solve the equation $\frac{1}{3}Bh = 64$. Of course, there are infinitely many possible solutions. The selection of values for the length, width, and height of the pyramid contributes to strengthening of students' number sense.

Figure 3: Cylinder with Volume of 64 cubic units



Formula for the volume of a cylinder: $V = Bh$. Given that the volume of the cylinder is 64 cubic units, students need to solve the equation $64 = Bh$. Again, there are infinitely many possible solutions. Students' choices for radius and height reveal their level of comfort and familiarity with operations involving decimal and irrational numbers.

The 'Starting with the answer' approach can be used for any mathematical idea that encompasses multiple applications of the same concept. Another example of using this approach to promote making sense of mathematics involves solving linear equations. Consider replacing numerous and varied examples of linear equations with Task 2.

3.1.2 Task 2 (Initial Questions)

The solution of a linear equation is 2.

- What is an example of such an equation?
- Is that equation unique? How do you know?

This task can be further differentiated and can target specific content standards by including additional questions.

3.1.3 Task 2 (Additional Questions)

The solution of a linear equation is 2. What is an example of such an equation if ...

- You need to use multiplication/division/addition/subtraction to solve it?
- You need to use a combination of multiple operations (be specific, if needed) to solve it?
- You need to apply distributive property to solve it?
- There is a variable on both sides of the equation?
- At least one coefficient is a fraction?

Students engaged in tasks that start with answers can work backward in any way they find appropriate and suitable to their knowledge and skill. The added benefit of this approach is that the stress of finding the correct answer has been removed from the very beginning.

3.2 Changing Perspectives

Have you ever been faced with a problem that went from extremely challenging to almost too simple once you changed your approach to solving it or a perspective from which you analyzed it? This phenomenon is common for puzzle lovers who sometimes need to step back from the problem at hand and look at it from a different angle (literally or figuratively). The Changing Perspectives approach requires students to do exactly this, in hopes that one of the alternative perspectives will aid them in their problem-solving process. This method is rooted in the effective teaching practices of using and connecting mathematical representations and selecting and implementing tasks that promote reasoning and problem solving. It may benefit not only students who struggle with more complex mathematical ideas (by allowing them to see the idea in a more concrete or visual world), but also students who easily learn new mathematical content. By representing the same idea using various representations, students, regardless of their mathematical ability or knowledge, create meaningful connections between concepts that may span over numerous units of study. These connections contribute to easier transfer and retention of the newly acquired knowledge.

To use the Changing Perspectives approach it is necessary to select a task that can be analyzed and represented in multiple ways. It is important to avoid procedural tasks and opt for rich problems that do not require the use of a certain problem-solving strategy. An example of such task and the accompanying questions and prompts are given in Table 1. The main benefit of using this approach is demonstrating to students that by making sense of problems one can answer mathematical questions, even if no formal mathematics is used in the problem solving process. When students, who are used to learning specific procedures for specific type of mathematical problems, fail to see a potential approach to solving a given problem, they rarely resort to making sense of it through pictures, models, or narrations. Instead, they ask for teacher or peer support, or give up. By incorporating different pathways to solutions within the given task, we communicate to students that they do not always need to have an equation or an expression to solve a mathematical problem, but they will always have to engage in thinking and sense making.

Table 1: Numerous Perspectives of a Task

Diane plans to use pieces of yarn that are $\frac{3}{4}$ feet long for her art project. How many of those pieces can she make if she has one piece of yarn that is 3 feet long?

1. Take a guess! How many pieces do you think Diane can make?
2. What mathematical sentence can describe the situation given in the problem?
3. Find a piece of yarn (or a string, or a long strip of paper). Model the situation using a ruler or a measuring tape. Take a picture of your model. What do you notice?
4. Draw a picture of the situation given in the problem.
5. Describe you know about the situation in your own words.
6. Based on what you have done, reflect on your initial guess. How good was it?

3.3 Choose Your Own Question

One of the biggest hurdles in supporting students in productive struggle is their own belief regarding mathematics. Due to years of traditional mathematics teaching and learning, many students have come to believe that the most important thing in solving mathematical problems is the answer, and that there is only one way to find that answer, typically the teacher's way. This belief may be detrimental for student success (especially in an online environment) if students do not immediately identify an appropriate approach to solving the problem. To remove this hurdle, consider transforming the problems into scenarios. To do so, start by selecting a problem that has multiple entry points, such as the one shown below.

T-shirt Problem

Students are hoping to raise money for personalized school t-shirts. The principal found a store that personalizes t-shirts and charges $10 for a shirt plus $2 for each letter. What algebraic expression can be used to represent the total cost of a shirt with n letters? How much would a shirt cost with your name on it?

Next, simply remove the question from the problem, effectively turning it into a scenario. Share the scenario with students and ask them what they notice about it. To ensure that the students spend adequate amount of time making sense of the given scenario, you may ask them to share certain number of noticings.

T-shirt Problem (Scenario Only)

Students are hoping to raise money for personalized school t-shirts. The principal found a store that personalizes t-shirts and charges $10 for a shirt plus $2 for each letter. What do you notice about this scenario? Share at least three things you notice.

Ask the students what they wonder about the scenario. Again, you may ask for several wonderings or you can differentiate the required number based on student need. Finally, students should select and solve at least one of their questions.

What do you wonder about this scenario? Share at least three things you wonder.
Choose one of your questions. Answer it using any approach you find appropriate.

The Choose Your Own Question approach is effective in activating students' prior knowledge, piquing their curiosity, and developing new knowledge by building on what they have already learned. The method embraces effective teaching practices, providing students with opportunities to raise questions that are personally relevant and meaningful.

3.4 Vehicle for Implementation of the Approaches

To ensure that the cognitive demand of tasks implemented using one of the above approaches remains at the desired level, it is necessary to provide opportunities for students to think about problems independently, share and discuss approaches with their peers, and engage in mathematical discourse. This may be achieved by incorporating the five practices for orchestrating productive mathematics discussion (Smith and Stein, 2011). Through the practices of anticipating, monitoring, selecting, sequencing, and connecting, teachers can incite and manage powerful student conversations and ideas exchange.

To efficiently do this in an online environment, technology tools, resources, and platforms such as online collaborative groups (for example, breakout rooms in Zoom or Google Meet), digital whiteboards (such as the whiteboard sharing tool in Zoom and Jamboard in G Suite), online collaborative tools (for example Padlet, Pear Deck, Google Docs, Slides, or Drawings) should be included in daily instruction.

4 Remembering What Matters amid the Pandemic

This too shall pass, but will the deviating from the effective teaching practices set us back years in math instruction? How far back do we have to go so we could start again in changing the face of mathematics teaching and learning? And what about all those learners who have been sitting in virtual classrooms and experiencing mathematics in a way that we know rarely leads to true comprehension and mastery? How big of an impact will these few of months of 2020 have on students' future math growth and achievement? Maybe, just maybe, we can do what we ask of our students every day, and persevere in preserving the integrity of our instruction regardless of the teaching modality by embracing the development of conceptual understanding, giving meaning to mathematical content, and empowering our students to grapple with mathematical ideas, rather than just algorithms. All it takes is being protective of our beliefs regarding mathematics education, a little bit of creativity, and a touch of bravery to embrace messiness of learning and extend making sense of problems to the online world of mathematics teaching and learning. *We have come so far. Let us not lose our way now.*

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